We have found a solution to the equation of motion. But is this the only one or are there more solutions? Also the sine function provides a valid solution. The most general solution is a linear combination of a sine and a cosine function

$$z = A\cos(\omega_0 t) + B\sin(\omega_0 t). \tag{2.6}$$

There is a more intuitive way to find the general solution. When we used the cosine function as solution, the oscillation started with the maximum extension at time zero. However, alternatively also any other time during the oscillation could be chosen as the start of the oscillation. This shift of the time corresponds to a shift of the phase of the oscillation by a constant phase shift ϕ. Thus all solutions are captured if the solution is shifted by a constant (but arbitrary) phase shift[2] ϕ, and the general solution results as

$$z = a\cos(\omega_0 t + \phi). \tag{2.7}$$

The two solutions given in (2.6) and (2.7) are in fact equivalent. Using the mathematical identity

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta, \tag{2.8}$$

the following relations between A, B in (2.6) and a, ϕ in (2.7) are obtained

$$A = a\cos\phi, \quad B = -a\sin\phi. \tag{2.9}$$