## 14.7 Dependence of the Phase on the Damping and on the Force Gradient

Generally, the dependence of the phase on the damping and on the force gradient is contained in (14.13). From Fig. 14.9, we can see that the dependence of the phase as function of frequency can be approximated as linear close to the resonance at $\omega = \omega_0$ or $\phi = -90°$. In the following, we will derive this linear relation between phase and frequency. Using in the nominator of (14.13), the approximation $\omega_0' \approx \omega_0$ and in the denominator the approximation $\omega_0' + \omega_0 \approx 2\omega_0$, as well as subsequently the relation $\delta\omega = \omega - \omega_0'$, results in

$$\tan\phi = \frac{-\omega\omega_0'}{Q_{\text{eff}}\left(\omega_0'^2 - \omega^2\right)} \approx \frac{-\omega_0^2}{Q_{\text{eff}}\left(\omega_0' + \omega\right)\left(\omega_0' - \omega\right)} \approx \frac{\omega_0}{2Q_{\text{eff}}\delta\omega} = \frac{k}{Q_{\text{eff}}k'}. \tag{14.14}$$

Close to the resonance, the phase will be close to $-\pi/2$ and the deviation from this value will be termed the phase shift $\Delta\phi$ with $\phi = -\pi/2 + \Delta\phi$. The arctan can be approximated in this case as $\arctan x \approx -\pi/2 - 1/x$, resulting in

$$\phi = -\frac{\pi}{2} + \Delta\phi = \arctan\left(\frac{\omega_0}{2Q_{\text{eff}}\delta\omega}\right) \approx -\frac{\pi}{2} - \frac{2Q_{\text{eff}}}{\omega_0}\delta\omega. \tag{14.15}$$

Thus the phase shift $\Delta\phi$ relative to the phase $-90°$ results as

$$\Delta\phi = -\frac{2Q_{\text{eff}}}{\omega_0}\delta\omega = \frac{2Q_{\text{eff}}}{\omega_0}\Delta\omega = \frac{Q_{\text{eff}}k'}{k} = -\frac{Q_{\text{eff}}}{k}\frac{\partial F_{\text{ts}}}{\partial z}\bigg|_{z=0}, \tag{14.16}$$

as the frequency $\delta\omega$ relative to the resonance frequency $\omega_0'$ corresponds to a shift of the resonance frequency by $-\Delta\omega$. This equation can be used for conversion between the frequency shift and the phase shift close to resonance. The phase shift depends linearly on both the effective quality factor and the force gradient of the tip-sample interaction. Since the phase depends on $\Delta\omega$ and $Q_{\text{eff}}$ in a different way than the amplitude, the phase recorded as a free signal (not used for the feedback) can result in a different contrast (phase contrast) than the amplitude signal.

According to (14.16), the sign of the force gradient determines the sign of the phase shift, since $Q_{\text{eff}}$ is always positive. For attractive forces (more precisely, positive force gradients) the phase is more negative than $-90°$ ($\phi < -90°$), and correspondingly for repulsive forces (more precisely, negative force gradients) the relation $\phi > -90°$ holds for the phase.