Section 162.2 page 278 starting three lines after Eq. (16.33)

(revised text)

negative phase voltage.). The working frequency ω_{work} is the frequency of the free cantilever plus the frequency shift $\Delta\omega$.

Now we discuss the frequency tracking capability of the PLL. For the moment, we do not yet consider the PI controller shown in Fig. 16.7 and assume that the phase signal V_{phase} is directly fed into the input of the VCO, i.e. $V_{\delta\omega} = V_{\mathrm{phase}}$. Let us assume that initially the frequency of the VCO matches the oscillation frequency of the cantilever, $\omega_{\mathrm{vco}} = \omega_{\mathrm{cant}} = \omega_{\mathrm{work}}$ (i.e. $\delta\omega = 0$) and $\phi_0 = +90°$ (i.e. $V_{\mathrm{phase}} = 0$) . In this case also the input voltage at the VCO vanishes, i.e. $V_{\delta\omega} = 0$.

Now we consider an increase of the actual oscillation frequency of the cantilever ω_{cant}, to ω'_{cant} by $\delta\omega$, e.g. due to a change in the tip-sample interaction. According to (16.32) this increase of the frequency by $\delta\omega$ leads to a phase signal measured by the phase detector $V_{\mathrm{phase}} \approx -K_{\mathrm{pd}}\delta\omega t$, which evolves linearly with time. With this input, the output frequency of the VCO increases according to (16.33) and since $V_{\delta\omega} = V_{\mathrm{phase}}$ as

$$\omega_{\mathrm{vco}} = \omega_{\mathrm{work}} - K_{\mathrm{pd}}K_{\mathrm{vco}}\cos(\delta\omega t + \phi_0) \approx \omega_{\mathrm{work}} + K_{\mathrm{pd}}K_{\mathrm{vco}}\delta\omega t. \qquad (16.34)$$

According to (16.34), a linearly increasing phase $\delta\omega t$ leads to an increasing ω_{vco}. This reduces the frequency difference $\delta\omega$ between the cantilever frequency and the frequency of the VCO. For a varying $\delta\omega(t)$ the term $\delta\omega t$ in (16.34) should be replaced by the integral $\int \delta\omega(t)dt$. The closer ω_{vco} comes

to ω'_{cant} (decreasing $\delta\omega(t)$), the smaller is the contribution to the integral becomes. Any remaining finite frequency mismatch $\delta\omega(t)$ leads over time to an increasing contribution to the integral, bringing the VCO frequency closer to ω'_{cant}. If ω_{vco} has reached ω'_{cant}, $\delta\omega(t)$ vanishes and the integral stays constant. In this way, the VCO frequency adapts to the increased frequency of the cantilever $\omega_{\mathrm{work}} + \delta\omega$. Due to this mechanism the VCO frequency is said to be locked to the cantilever frequency. In the steady-state $\omega_{\mathrm{vco}} = \omega_{\mathrm{cant}}$ and the frequency mismatch $\delta\omega(t)$ vanishes.

In the terminology of the PLL: The VCO frequency is *locked* to the cantilever oscillation frequency by a *phase* comparison of both signals in a feedback *loop*. Hence, the name *phase-locked loop*. In this way, the PLL measures the frequency deviation of the AFM sensor from ω_{work} as the voltage $V_{\delta\omega}$. This voltage, which is proportional to the frequency shift $\delta\omega$, is then used in the z-feedback loop to control the tip-sample distance and brings ω_{cant} back to ω_{work}. A certain deviation of the tip-sample distance from the setpoint value, corresponds to a certain frequency shift voltage $V_{\delta\omega}$, which is kept constant by the z-feedback loop (Fig. 16.5).

The original cantilever signal is a high-frequency signal close to ω_0, which is modulated to slightly lower or higher frequencies $\omega_0 + \delta\omega$ at a much lower frequency by the tip-sample interaction, for instance during scanning of an atomic corrugation (yet without z-feedback). The PLL converts (demodulates) this modulated high frequency signal to a low frequency signal proportional to the frequency modulation of the high frequency signal. This is called FM demodulation and also occurs in an FM radio receiver, where a high-frequency carrier signal is modulated by a low-frequency audio signal and the demodulation of the audio signal is desired.

Up to now we have concentrated on the frequency tracking capability of the PLL, while we turn now to the offset phase difference ϕ_0. Remember, that two frequencies are the same, if the relative phase is *constant*, not necessarily $+90°$. If the frequency of the VCO has adapted completely to the cantilever frequency, a constant phase offset ϕ_0 given by the integral $\int \delta\omega(t)dt$ (different from $+90°$), is present between both oscillations. However, the value $\phi_0 = +90°$ is desired, because it leads to maximum sensitivity of the phase detector.

The PI controller enforces a vanishing V_{phase} signal, by its setpoint value $V_{\text{phase}} = 0$. The PI controller controls the offset phase ϕ_0 to the desired value of $+90°$ between V_{cant} and V_{vco} by generating an appropriate controller output signal $V_{\delta\omega}$.